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# Dynamical Casimir effect with semi-transparent mirrors, and cosmology\*

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## Abstract

After reviewing some essential features of the Casimir effect and, specifically, of its regularization by zeta function and Hadamard methods, we consider the dynamical Casimir effect (or Fulling–Davies theory), where related regularization problems appear, with a view to an experimental verification of this theory. We finish with a discussion of the possible contribution of vacuum fluctuations to dark energy, in a Casimir-like fashion, that might involve the dynamical version.

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## 1. Introduction

It was observed some time ago that the universe expansion accelerates. This important discovery is still in search of an explanation. It could in fact be found within Einsteinian gravity, even if the only possibility there seems to be to consider the contributions of the quantum vacuum fluctuations of the fields pervading the universe to the cosmological constant, as was discussed by Zeldovich in a quite convincing way many years before the acceleration of that expansion was discovered [1]. This would be nice and, in principle, requires no new physics, however there are several problems, such as (i) the *cosmological constant problem*, that is, the contribution of the vacuum fluctuations seems to be exceedingly large, as compared with the mentioned astrophysical observations and (ii) the *coincidence problem*, related to the fact that in relative terms the associated energy is, in the present epoch, such a large part (over 72%) of the whole energy content of the universe (that is, of the same order of magnitude and even dominating the energy content of the universe). If we pay the price to modify Einstein's theory, then things become easier to adjust, but other problems emerge. One cannot be happy

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with such a number of possibilities, with increasing numbers of parameters, and many are just effective or phenomenological models: tensor, scalar–tensor, phantom, etc.

Here we will discuss a couple of specific problems of the vacuum fluctuations approach only, some of rather technical, the other of more fundamental nature, in relation to the regularization of quantum field theories in the presence of boundaries and with the so-called *dynamical Casimir effect*. We will recall a piece of sound mathematics needed for the regularization issue. Then we will explicitly illustrate an important aspect of this issue, namely, the introduction of physically meaningful regularization parameters, for the case of the dynamical Casimir effect. We will finish with a discussion of possible cosmological imprints of the Casimir effect, in some particular models.

## 2. Casimir effect: on the boundary divergences

Imposing mathematical boundary conditions on physical quantum fields is not easy, as first discussed by Deutsch and Candelas [2], who quantized EM and scalar fields near a smooth boundary and calculated the renormalized vacuum expectation value of the stress–energy tensor to find that the energy density *diverges* as the boundary is approached. Regularization and renormalization did not seem to cure the problem with infinities in this case, and an infinite *physical* energy was obtained if the mathematical boundary conditions were to be fulfilled. In an attempt to solve this, the authors argued that physical surfaces have nonzero depth, and this could be taken as a dimensional cutoff to regularize the infinities. Later, Kurt Symanzik did a rigorous analysis of quantum field theory in the presence of boundaries [3]. Prescribing the value of the quantum field on a boundary means using the Schrödinger representation, and Symanzik was able to show it to exist to all orders in the perturbative expansion. The issue was proven to be meaningful within the domains of renormalized quantum field theory. In this case, the boundary conditions and the hypersurfaces themselves were treated at a pure mathematical level (zero depth) by using delta functions.

New approaches to the problem have been postulated recently (see, e.g., [4]). Boundary conditions on a field,  $\phi$ , are enforced on a surface,  $S$ , by introducing a scalar potential,  $\sigma$ , of Gaussian shape living on and near the surface. When it becomes a delta function, the boundary conditions (Dirichlet here) are enforced: the delta-shaped potential kills *all* the modes of  $\phi$  at the surface. For the rest, the quantum system undergoes a full-fledged quantum field theory renormalization, as in the case of Symanzik’s approach. The results obtained confirm [2] in the several models studied but do not agree with [3]. They are also in contradiction to many textbooks and review articles dealing with the Casimir effect [5], where no infinite energy density when approaching the Casimir plates had been reported.

In some circumstances, specific regularization methods have been employed with success, as zeta function [6] and Hadamard regularization, this last in higher-post-Newtonian general relativity [7] and in recent variants of axiomatic and constructive quantum field theory [8]. Among mathematicians, Hadamard regularization is a rather standard technique in order to deal with singular differential and integral equations with boundary conditions, both analytically and numerically (for a sample of references, see [9]). Indeed, Hadamard regularization is a well-established procedure in order to give sense to infinite integrals [10]. Hadamard convergence is also one of the cornerstones in the rigorous formulation of quantum field theory through micro-localization, considered by specialists to be the most important step towards the understanding of linear partial differential equations since the invention of distributions (for a beautiful, updated treatment of Hadamard’s regularization, see [11]). In [10], the Hadamard regularization was invoked in order to fill the gap between the infinities appearing in the quantum field theory renormalized results and the finite values obtained in the literature with

other procedures. It was seen that the finite results derived using Hadamard's regularization coincide with the values obtained using the more classical and less rigorous methods in the literature on the Casimir effect. Moreover, Hadamard's prescription is able to separate and identify the singularities as physically meaningful cutoffs. Although the strict significance of this additional regularization is still not well understood, the fact that it is able to bridge the two approaches is already remarkable. In the following section, we present a case in a much related situation which can also serve as an example of the regularization issue in the Deutsch–Candelas fashion: we will clearly prove the advantages of using a prescription that, even if mathematical in nature, is very well adapted to proposed laboratory experiments.

### 3. The dynamical Casimir effect (Davies–Fulling)

The Davies–Fulling model [12, 13] describes the creation of massless particles by a moving perfect mirror following a prescribed trajectory. This phenomenon is also termed the *dynamical Casimir effect*. Moving mirrors modify the structure of the quantum vacuum, what manifests in the creation and annihilation of particles. Once the mirrors return to rest, a number of produced particles will generically still remain and can be interpreted as radiated particles. This flux has been calculated in the past in several situations by using different methods, as averaging over fast oscillations [14, 15], by multiple scale analysis [16], with the rotating wave approximation [17], with numerical techniques [18] and others [19]. Here we are interested in the production of the particles and their possible energy values while the mirrors are in movement. This is in no way a simple issue and a number of problems have recurrently appeared in the literature when trying to deal with it. To start, it is in this case far from clear which is the appropriate regularization to use. Different authors tend to employ different prescriptions, forgetting sometimes about the need to carry out a proper (physical) renormalization procedure, as was also the case in the other situations described in the preceding section. Thus, it turns out that ordinarily, in the case of a single, perfectly reflecting mirror, the number of produced particles as well as their energies diverge all the time while the mirrors move. Several prescriptions have been used in order to obtain a well-defined energy, however, for some trajectories this finite energy is *not* a positive quantity and cannot clearly be identified with the energy of the produced particles (see, e.g., [12]).

The approach I will describe here is a joint work with Haro [20], and relies on two very basic ingredients. First, the proper use of a Hamiltonian method and, second, the introduction of partially transmitting mirrors, which become transparent to very high frequencies. We have been able to prove in this way, both that the number of created particles remains finite and also that their energies are always positive, for the whole trajectories corresponding to the mirrors' displacement. We have also calculated from first principles the radiation–reaction force that acts on the mirrors owing to the emission and absorption of the particles, and which is related to the field's energy through the ordinary energy conservation law. As a consequence, the energy of the field at any time  $t$  is seen to equal, with the opposite sign, the work performed by the reaction force up to this time  $t$  [21, 22]. Such force is usually split into two parts [23, 24]: a dissipative force whose work equals minus the energy of the particles that remain [21], and a reactive force, which vanishes when the mirrors return to rest. It can be seen that the radiation–reaction force calculated from the Hamiltonian approach for partially transmitting mirrors satisfies, at all times during the mirrors' oscillation, the energy conservation law and can naturally account for the creation of positive energy particles. Also, the dissipative part obtained within this procedure agrees with the one calculated by other methods, as using the Heisenberg picture or other effective Hamiltonians (but those methods have traditionally

encountered problems with the reactive part, which in general yields a non-positive energy that cannot be considered as that of the particles created at any specific  $t$ ).

#### 4. A consistent formulation: semi-transparent mirrors

One of the main ingredients of the method is to use partially transmitting mirrors, which become transparent to very high frequencies (this is given by an *analytic matrix*). The second main ingredient is the proper use of a *Hamiltonian method* and the corresponding renormalization. We proved both that the number of created particles is *finite* and that their energy is always *positive*, for the whole trajectory during the mirrors' displacement. The radiation–reaction force acting on the mirrors owing to emission–absorption of particles is related to the field's energy through the ordinary energy conservation law: the energy of the field at any time  $t$  equals (with opposite sign) the work performed by the reaction force up to this time  $t$ . Such force is split into two parts: a *dissipative* force whose work equals minus the energy of the particles that remain and a *reactive* force vanishing when the mirrors return to rest. The dissipative part obtained agrees with the corresponding one from other methods. But those have problems with the *reactive* part, which in general yields a *non-positive* energy (which is not our case). To be noticed is that several proposals at an experimental verification of the dynamical Casimir effect have been issued recently.

##### 4.1. Some details and examples

We use a Hamiltonian method for a neutral Klein–Gordon field in a cavity  $\Omega_t$ , with boundaries moving at a certain speed  $v \ll c$ ,  $\epsilon = v/c$  (of order  $10^{-8}$  in the experimental proposal [25]). Assume the boundary is at rest for time  $t \leq 0$  and returns to its initial position at time  $T$ . The Hamiltonian density is conveniently obtained using the method in [26]. The Lagrangian density of the field is

$$\mathcal{L}(t, \mathbf{x}) = \frac{1}{2}[(\partial_t \phi)^2 - |\nabla_{\mathbf{x}} \phi|^2], \quad \forall \mathbf{x} \in \Omega_t \subset \mathbb{R}^n, \quad \forall t \in \mathbb{R}. \quad (1)$$

Transform now the moving boundary into a fixed one by the (non-conformal) change of coordinates

$$\mathcal{R}: (\bar{t}, \mathbf{y}) \rightarrow (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y})), \quad (2)$$

which converts  $\Omega_t$  into a fixed domain  $\tilde{\Omega}$ :  $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$  (with  $\bar{t}$  the new time).

The Hamiltonian density is

$$\tilde{\mathcal{H}}(\bar{t}, \mathbf{y}) = \frac{1}{2}(\tilde{\xi}^2 + J|\nabla_{\mathbf{x}} \tilde{\phi}|^2) + \tilde{\xi}(\partial_{\bar{t}} \tilde{\phi} - \sqrt{J} \partial_t \phi), \quad (3)$$

being  $\tilde{\phi}$  the field,  $\tilde{\xi}$  the conjugate momentum and  $J$  the Jacobian of the change  $d^3 \mathbf{x} \equiv J d^3 \mathbf{y}$ . It turns out that

$$\mathcal{H}(t, \mathbf{x}) = \mathcal{E}(t, \mathbf{x}) + \xi(t, \mathbf{x}) \langle \partial_s \mathbf{R}(\mathcal{R}^{-1}(t, \mathbf{x})), \nabla_{\mathbf{x}} \phi(t, \mathbf{x}) \rangle + \frac{1}{2} \xi(t, \mathbf{x}) \phi(t, \mathbf{x}) \partial_s (\ln J)|_{\mathcal{R}^{-1}(t, \mathbf{x})}. \quad (4)$$

As a simple example, for a single mirror following the prescribed trajectory  $R(\bar{t}, y) = y + \epsilon g(\bar{t})$ , we explicitly get

$$\mathcal{H}(t, x) = \mathcal{E}(t, x) + \epsilon \dot{g}(t) \xi(t, x) \partial_x \phi(t, x). \quad (5)$$

#### 4.2. Case of a single, partially transmitting mirror

In the original Davies–Fulling model [12], the renormalized energy is negative while the mirror moves; it cannot be considered as the energy of the produced particles at time  $t$  [cf paragraph after equation (4.5)]. An interpretation of this fact is that a perfectly reflecting mirror is non-physical. One should consider, instead, a *partially transmitting mirror*, transparent to high frequencies, what is indeed a mathematical implementation of a physical plate, continuing our discussion in the preceding section.

Consider the trajectory  $(t, \epsilon g(t))$ . When the mirror is at rest, *scattering* is described by the matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega) e^{-2i\omega L} \\ r(\omega) e^{2i\omega L} & s(\omega) \end{pmatrix}. \quad (6)$$

This  $S$  matrix is taken to be ( $x = L$  position of the mirror) (i) real in the temporal domain:  $S(-\omega) = S^*(\omega)$ , (ii) causal:  $S(\omega)$  is analytic for  $\text{Im}(\omega) > 0$ , (iii) unitary:  $S(\omega)S^\dagger(\omega) = \text{Id}$  and (iv) the identity at high frequencies:  $S(\omega) \rightarrow \text{Id}$ , when  $|\omega| \rightarrow \infty$ ,  $s(\omega)$  and  $r(\omega)$  being *meromorphic* (cutoff) functions: the material's *permittivity* and *resistivity*, respectively.

The results obtained are rewarding, and we can clearly see the origin of the divergence, in the perfect boundary conditions case, and its simple cure obtained in the semi-transparent mirror case. In fact, in this Hamiltonian approach the obtained force is

$$\begin{aligned} \langle \hat{F}_{\text{Ha}}(t) \rangle &= -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega \omega'}{\omega + \omega'} \text{Re} [e^{-i(\omega+\omega')t} \widehat{g}\widehat{\theta}_t(\omega + \omega')] \\ &\quad \times [ |r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 ] + \mathcal{O}(\epsilon^2), \end{aligned} \quad (7)$$

the integral *diverges* for a perfect mirror ( $r \equiv -1$ ,  $s \equiv 0$ , ideal case), but *nicely converges* for our partially transmitting (physical) one, where  $r(\omega) \rightarrow 0$  and  $s(\omega) \rightarrow 1$  as  $\omega \rightarrow \infty$ . Energy conservation is fulfilled: the dynamical energy at any time  $t$  equals, with the opposite sign, the work performed by the reaction force up to that time  $t$

$$\langle \hat{E}(t) \rangle = -\epsilon \int_0^t \langle \hat{F}_{\text{Ha}}(\tau) \rangle \dot{g}(\tau) d\tau. \quad (8)$$

The case of two partially transmitting mirrors is not so different. A similar, albeit more involved analysis, can be carried out [20]. No basic obstruction is envisaged to extend our procedure to higher dimensions and fields of any kind.

### 5. Cosmological imprint of the Casimir effect?

Although we still seem far from having an idea of what a fully-fledged theory of quantum gravity will look like in the end, semi-classical approaches to this issue led to the seminal idea, first clearly stated by Zeldovich [1], that quantum vacuum fluctuations, as a form of energy, must ‘gravitate’, that is, must enter into the vacuum expectation value of the stress–energy tensor  $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$  on the rhs of Einstein’s equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu}). \quad (9)$$

There may be subtleties in this argumentation, e.g., those pertaining to the following question: do quantum vacuum fluctuations fulfil the equivalence principle of general relativity? This seems to have been settled down very recently [27], but there are still contradictory answers in the literature [28]. This will affect *cosmology*, since  $\tilde{T}_{\mu\nu}$  excitations above the vacuum are in fact equivalent, in a given time slice, to a *cosmological constant*  $\Lambda = 8\pi G\mathcal{E}$ .

Recent observations yield the value [29]

$$\Lambda_{\text{obs}} = (2.14 \pm 0.13 \times 10^{-3} \text{eV})^4 \sim 4.32 \times 10^{-9} \text{erg cm}^{-3}. \quad (10)$$

As we said, the cosmological constant gets contributions from zero point fluctuations [1]

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \quad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\Lambda. \quad (11)$$

But evaluating in a box and putting a cutoff at maximum  $k_{\text{max}}$  corresponding to reliable quantum field theory physics (e.g. the Planck energy), one immediately gets the very huge number

$$\rho \sim \frac{\hbar k_{\text{Planck}}^4}{16\pi^2} \sim 10^{123} \rho_{\text{obs}}. \quad (12)$$

This is possibly the largest discrepancy between the theory and observation ever encountered in physics.

Assuming one will be able to prove that the ground value of the cc is *zero*, we will be left with this *incremental* value coming from the topology or boundary conditions. This sort of two-step approach to the cosmological constant is becoming very popular recently as the most accessible way to try to solve this extremely difficult issue [30]. We have then to see, using different examples, if this value acquires the correct order of magnitude—corresponding to the one coming from the observed acceleration in the expansion of our universe—under some reasonable conditions. The idea is to involve the *global* topology of the universe [31], in connection with the possibility that a faint scalar field pervading the universe could exist. Fields of this kind are ubiquitous in inflationary models, quintessence theories and the like. Also, given the fact that the universe expands, it is plausible that the dynamical Casimir effect should play a role in this discussion. Actually, one does not pretend in this way to solve the old problem of the cosmological constant, not even to contribute significantly to its understanding, but just to present simple and usual models which show that the right order of magnitude of (some contributions to)  $\rho_V$  which lie in the precise range deduced from the astrophysical observations may be not difficult to get. In different words, we only address here the ‘second stage’ of what has been termed by Weinberg [32] the *new* cosmological constant problem. It should be mentioned however, in this context, that there are some authors saying that the old cosmological constant problem is in fact trivial (see, e.g., [33]). That in any emergent gravity theory the natural value of the energy of the perfect non-perturbed vacuum (and thus of the cosmological constant) is exactly zero. The small, nonzero value that we see just comes from perturbations of the vacuum, due to the expansion of the universe, gravitating matter and effects of boundaries as discussed below.

### 5.1. Simple model with large and small compactified dimensions

We assumed the existence of a scalar field extending through the universe and calculated the contribution to the cosmological constant from the Casimir energy density of this field, for some typical boundary conditions. Ultraviolet contributions must be safely set to zero by some mechanism of a fundamental theory. Another hypothesis is the existence of both large and small dimensions (the total number of large spatial coordinates being always three), some of which may be compactified, so that the global topology of the universe plays an important role. There is a quite extensive literature both in the subject of what is the global topology of spatial sections of the universe [31] and also on the issue of the possible contribution of the Casimir effect as a source of some sort of cosmic energy, as in the case of the creation of a neutron star [34]. There are arguments that favor different topologies, as a compact hyperbolic manifold for

the spatial section, what would have clear observational consequences [35]. Other interesting work along these lines was reported in [36] and related ideas have been discussed very recently in [37]. However, we differ from all those in that we put emphasis just in obtaining the right order of magnitude for the effect. At the present level, it has no sense yet to consider more specifications concerning the nature of the field, the different models for the topology of the universe and the different boundary conditions possible, with its effect on the sign of the force too. This is left to future analysis. From previous results [38], we know that the range of orders of magnitude of the vacuum energy density for the most common possibilities is not so widespread, and may only differ by at most a couple of digits. This allows us, both for the sake of simplicity and universality, to deal with two simple situations, corresponding to a scalar field with periodic boundary conditions or spherically compactified. As explained in [39], most cases with usual boundary conditions reduce to those, from a mathematical viewpoint.

For lack of space we will not describe these models in detail here (this has been done elsewhere [40]). Suffice to say that it can be proven that the contribution of the vacuum energy of a small-mass scalar field, conformally coupled to gravity, and coming from the compactification of some small (2 or 3) and some large (1 or 2) dimensions—with compactification radii of the order of 10–1000 the Planck length in the first case and of the order of the present radius of the universe, in the second—lead to values that compare well with observational data, in order of magnitude, with the exception of the *sign*—which turns out to be opposite to the one needed to explain negative pressure. To deal with this crucial issue, we consider the two following classes of models.

### 5.2. Braneworld models

Braneworld theories may hopefully solve both the hierarchy problem and the cosmological constant problem. The bulk Casimir effect can play an important role in the construction (radion stabilization) of braneworlds. We have calculated the bulk Casimir effect (effective potential) for conformal and massive scalar fields [41]. The bulk is a five-dimensional AdS or dS space, with 2 (or 1) four-dimensional dS branes (our universe). The results obtained are quite consistent with the observational data.

### 5.3. Supergraviton theories

We have also computed the effective potential for some multi-graviton models with supersymmetry [42]. In one case, the bulk is a flat manifold with the torus topology  $\mathbb{R} \times \mathbb{T}^3$ , and it can be shown that the induced cosmological constant can be rendered *positive* due to topological contributions [43]. Previously, the case of  $\mathbb{R}^4$  had been considered. In the multi-graviton model the induced cosmological constant can indeed be positive, but only if the number of massive gravitons is sufficiently large, what is not easy to fit in a natural way. In the supersymmetric case, however, the cosmological constant turns out to be positive just by imposing anti-periodic boundary condition in the fermionic sector. An essential issue in our model is to allow for non-nearest-neighbor couplings.

For the torus topology we have got the topological contributions to the effective potential to have always a fixed sign, which depends on the boundary condition one imposes. They are negative for periodic fields, and positive for anti-periodic ones. But topology provides then a mechanism which, in a natural way, permits to have a positive cosmological constant in the multi-supergravity model with anti-periodic fermions. The value of the cosmological constant is regulated by the corresponding size of the torus. We can most naturally use the minimum number,  $N = 3$ , of copies of bosons and fermions, and show that—as in the first, much more



simple example, but now with the right sign!—within our model the observational values for the cosmological constant can be approximately matched, by making quite reasonable adjustments of the parameters involved. As a byproduct, the results that we have obtained [43] might also be relevant in the study of electroweak symmetry breaking in models with similar type of couplings, for the deconstruction issue.

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